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Use of damage lines in predicting cumulative damage in fatigue

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USE OF DAMAGE LINES IN PREDICTING CUMULATIVE DAMAGE IN FATIGUE

WARREN R. COLEGROVE

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USE OF DAMAGE LINES IN PREDICTING CUMULATIVE DAMAGE IN FATIGUE

by

Warren R. Colegrove

Lieutenant, U. S. Navy

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Submitted in partial fulfillment
of the requirements
for the degree of
Master of Science

United States Naval Postgraduate School
Monterey, California
1953

C573

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

MECHANICAL ENGINEERING

from the

United States Nava l Postgraduate School

PREFACE

This study was performed during the period February through May 1953 in the Materials Testing Laboratory of the U. S. Naval Postgraduate School, Monterey, California. The work was undertaken in an effort to test a means of predicting cumulative fatigue life for machine parts subjected to varying loads.

The author wishes to acknowledge the helpful suggestions and efforts in the preparation of this study received from Dr. Robert E. Newton, Professor of Mechanical Engineering at the U. S. Naval Postgraduate School. In addition the author wishes to thank Mr. J. A. Octavec for the extreme care exercised in the preparation of the test specimens.

TABLE OF CONTENTS

PART	TITLE	PAGE
	Title Page	
	Certificate of Approval	i
	Preface	ii
	Table of Contents	iii
	List of Tables	iv
	List of Figures	v
	Table of Symbols and Abbreviations	vi
	Summary	1
I	Introduction	2
II	Materials, Method of Testing and Procedure	6
III	Results and Discussion	12
IV	Conclusions and Recommendations	19
	List of References	21
	Figures 2 - 9	
	Tables 1- 7	

LIST OF TABLES

1. Tests made to establish S-N curves.

2. Tests made to establish damage lines.

3. Tests made to determine accuracy of damage lines.

4. Standard deviation and mean log N obtained in establishing S-N curves.

5. Standard deviation and mean log N obtained in establishing "1/3" damage line.

6. Standard deviation and mean log N obtained in establishing "2/3" damage line.

7. Standard deviation and mean log N obtained in testing damage lines obtained.

LIST OF FIGURES

1. Plot showing damage lines on S-N diagram.

2. Sketch of test specimen employed.

3. Composite S-N curves for various probabilities of failure, P.

4. Plot showing "1/3" damage points and scatter obtained in establishing these points.

5. Plot showing "2/3" damage points and scatter obtained in establishing these points.

6. Plot showing scatter resulting in tests to check the accuracy of the "1/3" damage points at two stress levels.

7. Plot showing scatter resulting in tests to check the accuracy of the "2/3" damage points at two stress levels.

8 and 9. Plots showing scatter resulting in tests to check the accuracy of the damage points at three stress levels.

TABLE OF SYMBOLS AND ABBREVIATIONS

n_1, n_2, etc.	Cycles of stress at S_1, S_2, etc.
N_1, N_2, etc	Cycles of stress to failure on S-N diagram at S_1, S_2, etc.
n_1/N_1, n_2/N_2, etc . . .	Cycle ratio at S_1, S_2, etc.
$\Sigma\, n/N$	Cumulative cycle ratio or cumulative fatigue life.
n_p	Cycles of stress at Prestress
n_t	Cycles of stress at Test Stress

SUMMARY

The purpose of this study was to determine the feasibility of damage lines for predicting the cumulative fatigue life of machine parts subjected to varying loads. Four type SF-2, Sonntag Flexure Fatigue Machines were employed using type SF-2, specification #1, specimens. The specimens were manufactured from 24S-T4 0.065 inches thick sheet aluminum. The test results were analyzed statistically because of the large scatter normally encountered in fatigue testing.

The basic S-N curves were established with limits of scatter represented by probability limits of 90% determined from the standard deviation at the four stress levels used. Two damage lines were then established using the same methods as above and tested for reliability. Two hundred ninety-seven tests were conducted in all.

The results indicate that damage lines can be constructed on the S-N diagram with great time and cost involved. The tests to determine their accuracy showed them reasonably reliable. There was an indication that a simpler theory based on predicting the value of $\Sigma n/N$ equal to unity could be used for a portion of the stress spectrum for the aluminum tested.

CHAPTER I

Introduction

The study of the behavior of metals under load application of varying amplitude is an important problem facing the design engineer.

Since most fatigue tests are run under conditions of constant load amplitude during the application of cycles of stress, producing the familiar S-N diagram, the question is raised as to whether we can use the S-N diagram for predicting the life of a part subjected to varying loads.

One of the earlier investigations in this field was made by M. S. Miner, [2], in which he proposed that the cumulative fatigue life of a part under varying loads be computed using the following equation:

$$n_1/N_1 + n_2/N_2 + \cdots = \Sigma n/N = 1.0$$

where n_1 = number of cycles applied at stress S_1.

N_1 = number of cycles of fatigue life on S-N curve at stress S_1.

n_2 = number of cycles applied at Stress S_2.

N_2 = number of cycles of fatigue life on S-N curve at stress S_2.

n_1/N_1 = cycle ratio at S_1, n_2/N_2 = cycle ratio at S_2.

$\Sigma n/N$ = cumulative cycle ratio or cumulative fatigue life.

In the experiments carried out to support this hypothesis, Miner obtained an average value of cumulative cycle ratio equal to unity, however, his tests were too few in number to sustain the hypothesis. Subsequent to Miner's work many investigators have concluded that very often the individual results have been too far removed from unity to justify use of this hypothesis. A study of the work in this phase of fatigue seems to indicate that when the stress is reduced in a series of stages the value of $\Sigma n/N$ is less than unity. When the stress is increased in a series of stages the value of $\Sigma n/N$ is greater than unity.

Of the various hypotheses presented in this field, the one analyzed and tested by Newmark and Richart [4] seems to offer the most promise of overcoming the above mentioned shortcomings of Miner's hypothesis. In brief, the hypothesis is based on the assumption that the damage to a fatigue test specimen depends on the cycle ratio, n/N, but the dependence is different at different stress levels.

The implications of this statement can best be explained by use of Fig. 1 below.

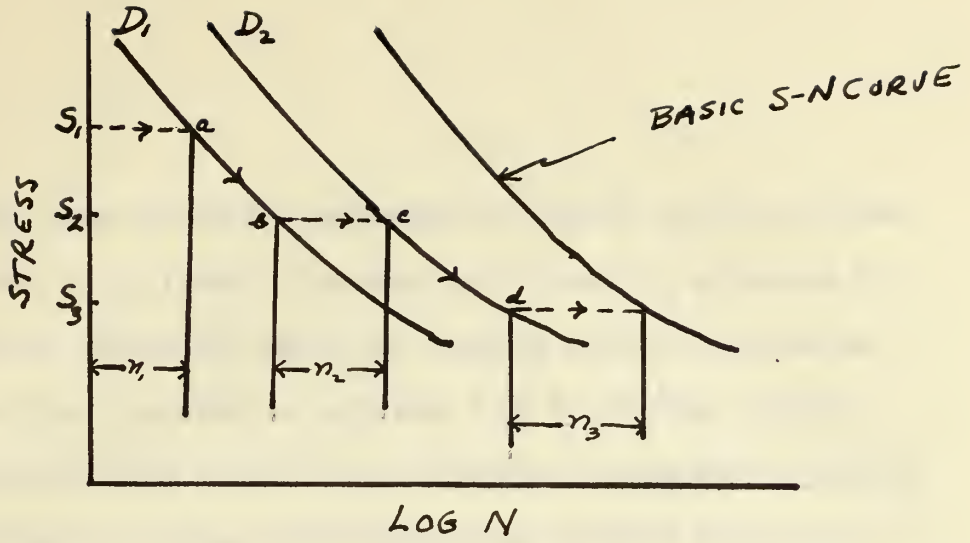

FIG 1. PLOT SHOWING DAMAGE LINES ON S-N DIAGRAM.

In this plot the basic S-N curve is supplemented by the curves D_1 and D_2 which are called constant damage lines. These constant damage lines are defined as the lines connecting points at different stress levels which have an equal "degree of damage". We note that the parameter assigned D_1 and D_2 must necessarily be arbitrary.

As can be seen by analyzing Fig. 1, damage lines constructed in accordance with Miner's hypothesis would consist of a series of S-N curves displaced to the left. The latter hypothesis mentioned, however, indicates that these lines will be skewed with reference to the basic S-N curve.

The method of obtaining cumulative effects at different stress levels is illustrated in Fig. 1.

Consider that we stress a specimen as follows; apply n_1 cycles of Stress S_1, n_2 cycles of stress S_2, followed by stressing to failure at stress S_3. Using the diagram, we first proceed at stress S_1 for n_1 cycles to the point a on D_1. We then proceed a long D_1 to point b, move out at constant stress S_2 for n_2 cycles to the point c on D_2, proceed along the constant damage line D_2 to point d a nd stress to failure at S_3. Theoretically there should be n_3 cycles remaining at stress S_3. The resulting value of cumulative cycle ratio is obtained by adding the individual cycle ra tios at the three stress levels. Newmark and Richart made a series of tests to verify this hypothesis but their tests were too few in number to validate it.

Past experimental evidence gathered on all types of fatigue testing indicates that the problem of fatigue is statistical in nature, hence any attempt to prove or disprove the hypothesis s hould be subjected to statistical methods. In view of this, it is the purpose of this paper to describe an experimental progra m carried out to determine the feasibility of the damage line hypothesis, using statistical methods of analysis.

CHAPTER II

Material, Method of Testing and Procedure.

The 24S-T4 0.065 inches thick sheet aluminum alloy used in the testing was obtained from standard Navy stock. It was packaged for shipment to prevent damage from handling. Standard 1 5/8 inch cantilever specimens, SF-2, specification #1, detailed in Fig. 2, were machined in the machine shop of the U. S. Naval Postgraduate School for use with four Sonntag Flexure Fatigue machines, type SF-2. The machines are constant repeated force fatigue machines using an eccentric mass to generate the force. The eccentricity of the mass is adjustable giving a maximum force P at the free end of the specimen according to the following formula for the specimen used:

$$\sigma = \frac{9.235 P}{h^2}$$

σ = STRESS AT OUTER FIBRES.
P = LOAD AT FREE END OF CANTILEVER.
h = SPECIMEN THICKNESS.

The machines were carefully tuned for natural frequency using the procedure as outlined in the manufacturer's instruction book.

The specimens were machined from the sheet stock so that the length of the cantilever was in the direction of rolling. This direction was chosen because previous tests by Oberg, T. T. and Rooney, R. J. [3] indicate that this direction yields the least

scatter in test results. Upon final machining the specimens were polished with 00 and 000 polishing paper to remove the oxide film and any machining scratches. Specimen thickness was measured using a micrometer and interpolating to the nearest ten-thousandth of an inch. Five specimens were rejected prior to testing. Specimens 36 and 55 were not used in establishing the S-N curves because the eccentric mass shifted during the run.

The S-N curves were established by conducting fourteen tests at each of four different stress levels, namely 39,400, 33,000, 29,600 and 25,400 psi. Frequency distribution diagrams of the number of cycles to failure, N, at a given stress level were skewed whereas the frequency distribution of log N assumed a more nearly normal shape. Therefore, for purposes of this paper the logarithmic - normal distribution was assumed. This procedure is in agreement with the majority of investigations of this nature, e.g. Epremian, E. and Mehl, R. F. [1] and Sinclair, G. M. and Dolan, T. J. [6] .

The fourteen tests at each stress level were analyzed using established methods for determining standard deviation as described by Scarborough, J. B. [5] . Using the standard deviation obtained, probability limits for 90% were obtained for representing the scatter at each of these stress levels. The results are plotted in Fig. 3 with P = 0.50 representing the mean value of these tests

and P= 0.05 and P = 0.95 representing the upper and lower limits of the 90% proba bality band. The standard deviations and mea n values of log N are listed in Table 4.

The next step in the procedure was to establish damage lines. The decision was made to use the stress levels 39,400, 33,000, and 29,600 psi for establishing these lines, inas much as time involved in testing at those levels would be less and would permit more thorough analysis. During the course of the test program it became apparent that the inclusion of the 25,400 psi level would be desirable. This stress was, therefore, included in the procedure during the latter stages of the program.

The first damage line was established by pre-stressing the specimens at 39,400 psi for 1/3 N, or 17,500 cycles, followed by testing to failure at the lower levels. Fourteen tests were conducted for each variation of this procedure. The damage points were obtained at each stress level by assuming logarithmic-normal distribution as before, and working back from the mean value of log N at each level with the values of cycles remaining at that stress after the prestress. The results of these tests were analyzed in the same manner as the first group of tests for obtaining standard deviation and the 90% probability limits. These results are plotted in Fig.4 with the probability limits

plotted as points superimposed on the original S-N band shown in Fig. 3 and reproduced in Fig. 4. (The original S-N band is reproduced similarly in Figs. 5, 6, 7, 8 and 9.) It is noted that these damage points represent 1/3 N only at the prestress level, 39,400 psi, therefore future reference to these damage points or the damage line will be enclosed in quotation marks, e.g., "1/3" damage line.

In a similar manner the second damage line was established by prestressing the specimens at 39,400 psi for 2/3 N or 35,000 cycles and testing to failure at the lower levels. These results are represented in Fig. 5. This line will be referred to as the "2/3" damage line. Individual test results for both the "1/3" and the "2/3" damage points are tabulated in Table 2. Standard deviation and mean log N are given in Tables 5 and 6 for these tests.

Having established the damage points, tests were then made to determine their validity when the prestress and the test stress were interchanged. For maximum testing it was decided to run only five specimens for each of the variations of this set. The following program was set up to include testing at two and three stress levels.

TWO STRESS LEVEL TESTS (80 TESTS)

Prestress to "1/3" Damage	Test Stress to Failure
33,000	39,400
33,000	29,600
33,000	25,400
29,600	39,400
29,600	33,000
25,400	39,400
25,400	33,000
25,400	29,600

Prestress to "2/3" Damage	Test Stress to Failure
33,000	39,400
33,000	29,600
33,000	25,400
29,600	39,400
29,600	33,000
25,400	39,400
25,400	33,000
25,400	29,600

THREE STRESS LEVEL TESTS (60 TESTS)

Prestress to "1/3" Damage	Test Stress to "2/3" Damage	Test Stress to Failure
39,400	33,000	39,400
39,400	33,000	29,600
39,400	29,600	39,400
39,400	29,600	33,000
33,000	39,400	33,000
33,000	39,400	29,600
33,000	29,600	39,400
33,000	29,600	33,000
29,600	39,400	33,000
29,600	39,400	29,600
29,600	33,000	39,400
29,600	33,000	29,600

The results of the individual tests are given in Table 3 and the limits of scatter are shown graphically in Figs. 6,7, 8, and 9 in a manner similar to that previously mentioned for the damage points. These tests were analyzed to determine standard deviation and the 90% probability limits as before. Tables 7 lists the values of standard deviation and mean log N obtained.

CHAPTER III

Results and Discussion

The results obtained from this study are subject to many types of errors. Some of the errors would be present in any fatigue testing, whereas others are peculiar to the type machines used and techniques used.

In the former category are non-homogeniety of the material and minor machining defects. In any metal it is impossible to obtain homogeniety of structure in sizes larger than single crystals of the metal. This is the largest single factor which necessitates handling fatigue results statistically. No control of machining defects was exercised other than the polishing technique previously mentioned.

Four type SF-2 Sonntag machines were available at the outset for testing, however, various troubles arose with Machines #1 and #3 which rendered them out of commission temporarily. Machine #1 was used throughout the majority of the testing. Only nine tests were performed on Machine #3. Machines #2 and #4 were used throughout the program and no difficulty was encountered with them. The results obtained from the latter two machines appeared to agree quite well whereas Machine #1 tended to give lower results. The use of four machines contributed to some of the scatter encountered but due to the low speed of this type machine multiple units were requisite to complete the testing program.

Some errors introduced by experimental procedure include inaccuracy in reading specimen thickness with the micrometer, inaccuracy in setting the correct load on the machine, bending the specimen while loading, bending produced while changing the load with a specimen installed, inability to read the counter on the machines closer than 1000 cycles, and the variables entering during the starting and stopping of the machines. For varying the stress levels in the two and three stress level programs it was necessary to stop and start the machine once and twice, respectively, and to reset the load each time. It is considered practically impossible to change the load with the specimen in place without producing some bending in the specimen. This was avoided in the one stress level tests by setting the load before installing the specimen. With care the specimens can be loaded without producing any bending by first clamping the end of the specimen to which the force is applied and then setting up on the "built-in" end.

The composite S-N curves obtained for P = 0.05, 0.50, and 0.95 are presented in Fig. 3. The width of the scatter band is determined by the standard deviation and, as can be noted by referring to Fig. 3, tends to broaden at the lower level of stress. The values of standard deviation agree reasonably well with other investigations made on aluminum and it is therefore felt that the scatter encountered was normal.

The damage points established for "1/3" and "2/3" damage are shown graphically in Figs. 4 and 5. These points were established using the same number of tests as for the basic S-N curves and they must be given equal validity to the S-N curves. As will be readily noted it would be very difficult to fair a curve through these points. Displacing B and B' slightly to the left for approximately 10,000 cycles and C and C' to the right about 15,000 cycles (not an inordinate amount considering the total N at these levels) would make possible a faired curve for establishing the complete damage lines. It is interesting to note that the scatter bands obtained with these tests are smaller than those resulting when the specimens were tested to failure at one stress level. This is to be expected since part of the specimens' life was "spent" at the high stress level where the scatter band was only 25,000 cycles in width. Conversely, when the low stress level is the prestress it is to be expected that the scatter band will be wider at the higher stress.

Another interesting feature of these damage points is that the points A, B, C and A', B', C' establish lines which closely parallel the S-N curves. This would indicate that for stress variation in this range the prediction by Miner's hypothesis of $\Sigma n/N = 1.0$ is reasonably accurate.

The tests at two and three stress levels to determine the validity of the damage points as established are shown graphically

in Figs. 6, 7, 8 and 9. The scatter of these tests is in the same general range as that previously determined. There is, however, a displacement of mean life. This might be expected since the exact location of the equal damage points is not known, and yet in these tests we have applied the various cycle blocks considering them as definite points on the S-N curve. The general trend was for the mean life to be displaced to the right on the S-N curve when going from a low stress to a higher stress. This was not true in all cases.

The values of cumulative cycle ratio tended to be lower for the three level tests than the two level tests. This could be attributable to the fact that the machines were stopped and started an additional time in the three level tests.

In Table 3 are listed the cumulative cycle ratio predicted and that actually obtained. The predicted ratio is based on the damage points and the mean life. A study of these values indicates that for the tests conducted in the range from 29,600 psi to 39,400 psi the predicted cycle ratio is no more accurate than the simpler prediction of Miner's, $\Sigma n/N = 1.0$. This indicates that if the stress spectrum for a given application has a short stress range, the hypothesis of Miner's is sufficient for predicting the cumulative cycle ratio.

The locations of points D and D' indicate severe damage at

25,400 psi from a prestress at 39,400 psi. The tests to check the validity of these points are not as numerous as is desirable due to the great amount of time required to make them. Of those tests made from this lower level the general tendency is to substantiate the prediction of cumulative cycle ratio greater than one. There are however, sufficient tests where the actual value of $\Sigma n/N$ is very close to unity to indicate that Miner's hypothesis would be a safer prediction.

The location of D and D' inside the 90% probability limits raises an interesting point. If the designer were to use these curves and desire to stay to the left of the P = 0.05 line, Figs. 2 and 3 would indicate that having prestressed a part to the 1/3 damage point at 39,400 psi, there would be no fatigue life remaining at the 25,400 psi level. And yet there would be some fatigue life remaining at the higher and hence more severe stresses of 29,600 psi and 33,000 psi. This seemingly anomalous condition is caused by the large scatter band in terms of cycles, N, at the 25,400 psi stress level. In testing from 1/3 N at 39,400 psi to this level the average life actually remaining at the lower level was approximately 200,000 cycles, with a low value of 129,000. This same condition prevails using the "2/3" damage line. This would indicate some conservatism in using the damage lines in predicting total life.

A check was made of the 140 tests listed in Table 3 to see

what the result would be if Miner's hypothesis were used to predict the $\Sigma n/N$, basing the prediction on the P = 0.05 S-N line. Seven of these tests (5%) fell on the low side of this prediction and in each of these cases the prediction was only slightly low. In contrast, where the damage line and mean life was used to predict $\Sigma n/N$, fifty-four tests (38.5%) fall on the unsafe side.

Due to the large number of specimens tested, the writer had occasion many times to watch the fracture develop in the specimen. The appearance of cracks on the upper surface of the specimens preceded complete fracture by a very few cycles of stress in the majority of cases. It was noted in some instances that a great number of cracks might develop on the surface before fracture. In the great majority of these cases the fatigue life of the specimen was very high when compared to those specimens undergoing similar stressing. In all cases where the crack density was great the specimen was being stressed at the lower levels of stress included in the study. This raises the question: "Do the cracks stress relieve one another?" It appears that the cracks might be so oriented as to bring about mutual stress relief and thus prolong life.

In view of the above a good portion of the scatter encountered in fatigue testing might be attributable to this stress relief phenomenon. If means were available for determining when the

first crack appeared, the scatter might be substantially reduced.

CHAPTER IV

Conclusions and Recommendations for Future Study

The following conclusions are drawn from the results obtained in this study.

1. For the aluminum alloy used, this study indicates that reliable constant damage lines can be obtained by the process of prestressing at a high stress level and testing to failure at lower levels.

2. In the stress range from 39,400 to 29,600 psi the damage lines closely parallel the S-N curve so that in this range the damage line hypothesis is substantially the same as the hypothesis of Miner. In this range there is no advantage in the use of damage lines.

3. The damage lines drawn to include the lower stress level, 25,400 psi., skew to the right with reference to the basic S-N curve in such a manner as to predict values of $\Sigma n/N$ less than unity when decreasing stress and greater than unity when increasing stress. The tests made including this lower level of stress show an overall tendency to substantiate this prediction. The tests were too few in number, however, for any definite conclusions to be drawn as to the feasibility of the damage lines in this lower band.

4. The two damage lines obtained in this work have a similar shape over the complete stress spectrum investigated. This indicates the possibility that a complete band of constant damage lines might be made by obtaining a low and high value damage line experimentally and constructing the intermediate lines by graphical interpolation.

The conclusion expressed under 2 above is very important from the designer's viewpoint. If he were fortunate enough to have a stress spectrum in the range from 39,400 to 29,600 psi., this study indicates that the best procedure open to him would be to use Miner's hypothesis basing his cycle ratios on the lower limit of the S-N band, or a curve such as P = 0.05. This would eliminate costly and time-consuming experiments to establish damage lines.

It is recommended that any future studies in this field should be pointed toward a program which will show conclusively the effects of the tendency of the damage lines to skew to the right at the lower levels.

With regard to the damage lines it is felt that the similarity of the two lines constructed in this study indicate a strong possibility that the damage lines follow a pattern that would allow graphical interpolation. This requires further proof, however, with a program set up primarily to test its feasibility.

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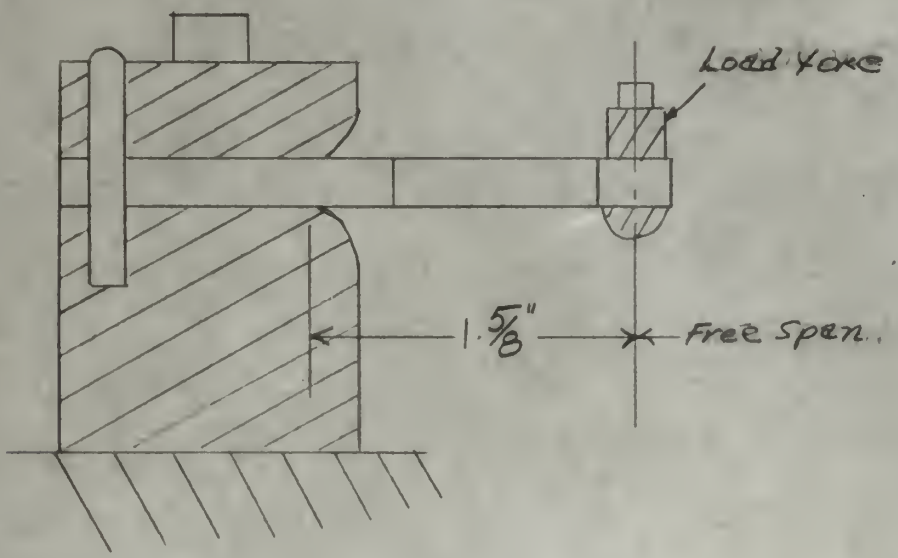
$1\frac{5}{8}$" SPECIMEN SF-2
SPEC #1



Mounting

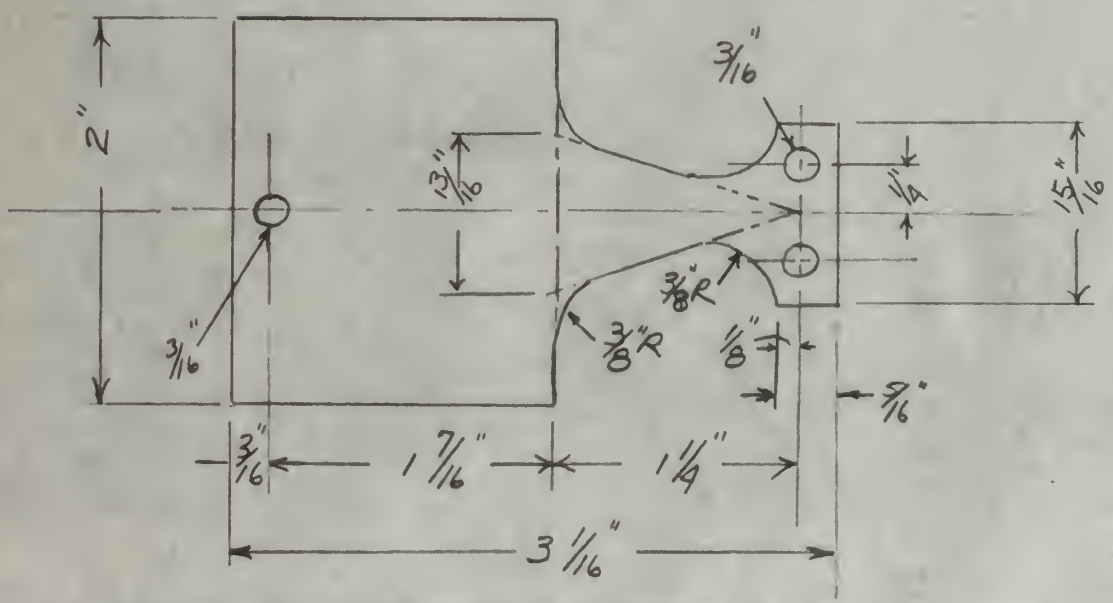


Fig 2. Showing Specimen and Mounting.

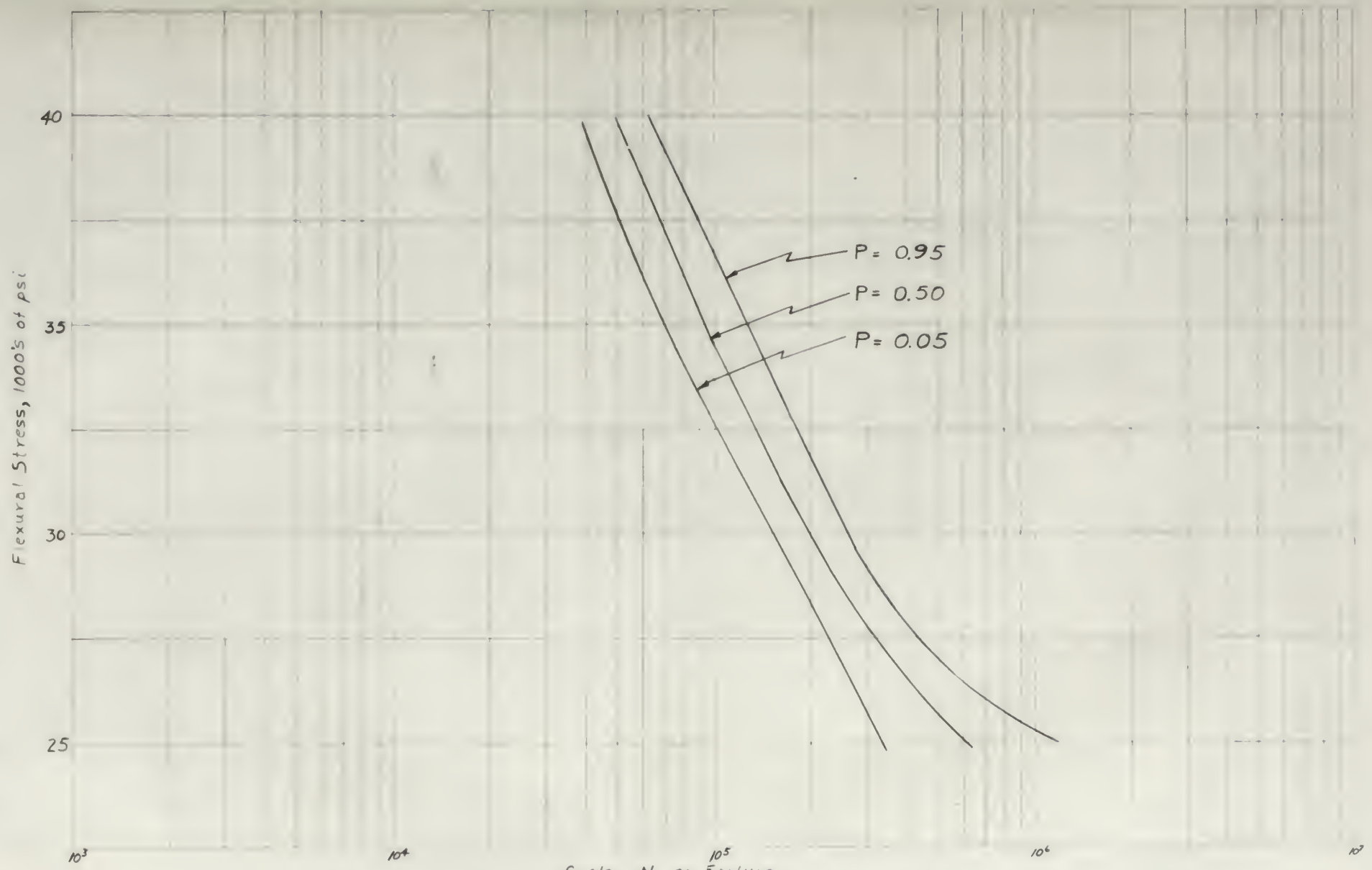


Fig 3 Composite S-N Curves for Various Probabilities of Failure, P.

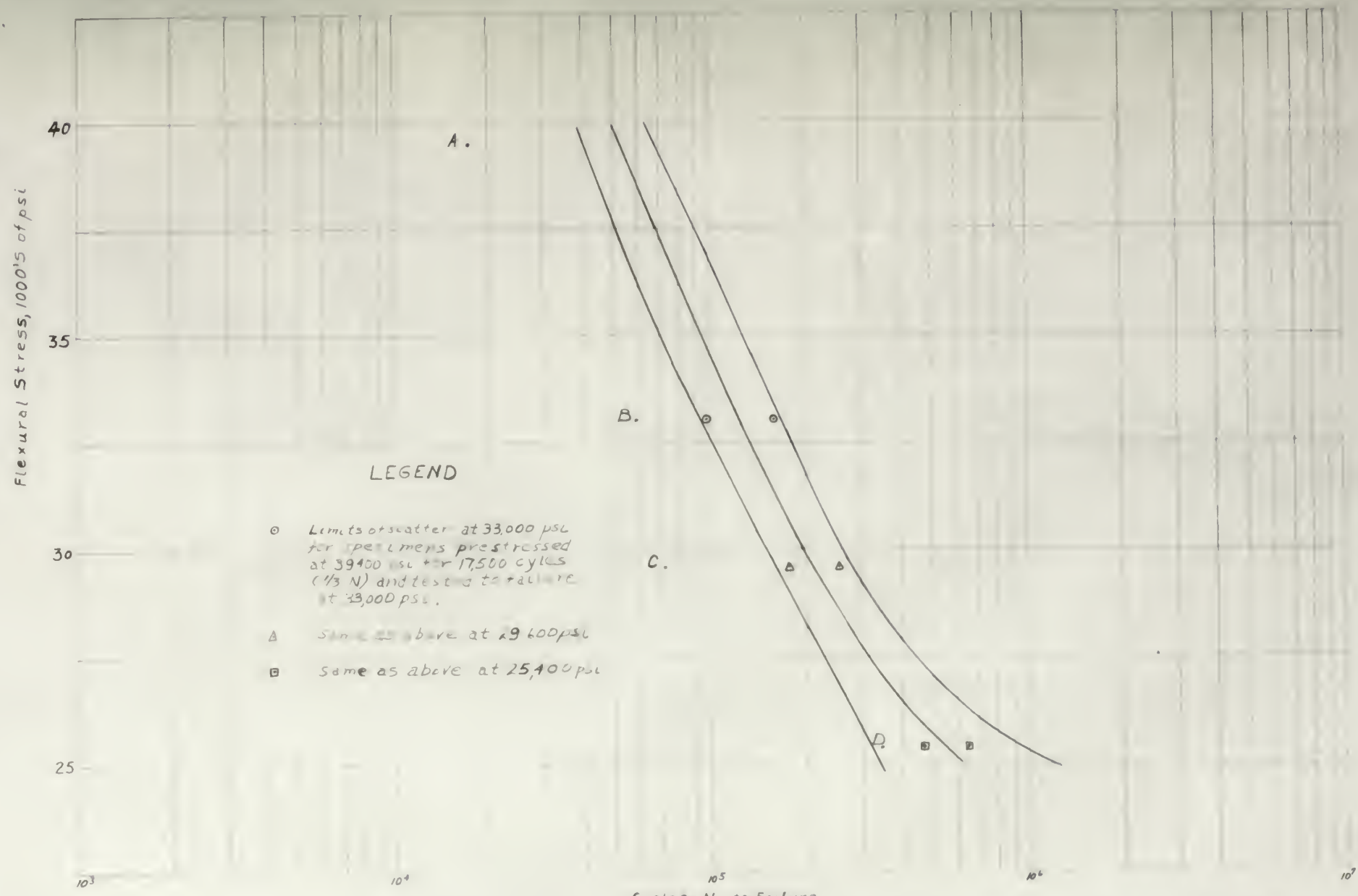


Fig 4. Plot Showing Damage Points at 33,000, 29,600 & 25,400 psi Corresponding to Damage at 1/3 Fatigue Life at 39,400 psi with Limits of Scatter from P=0.05 to P=0.95. (14 tests at each level)

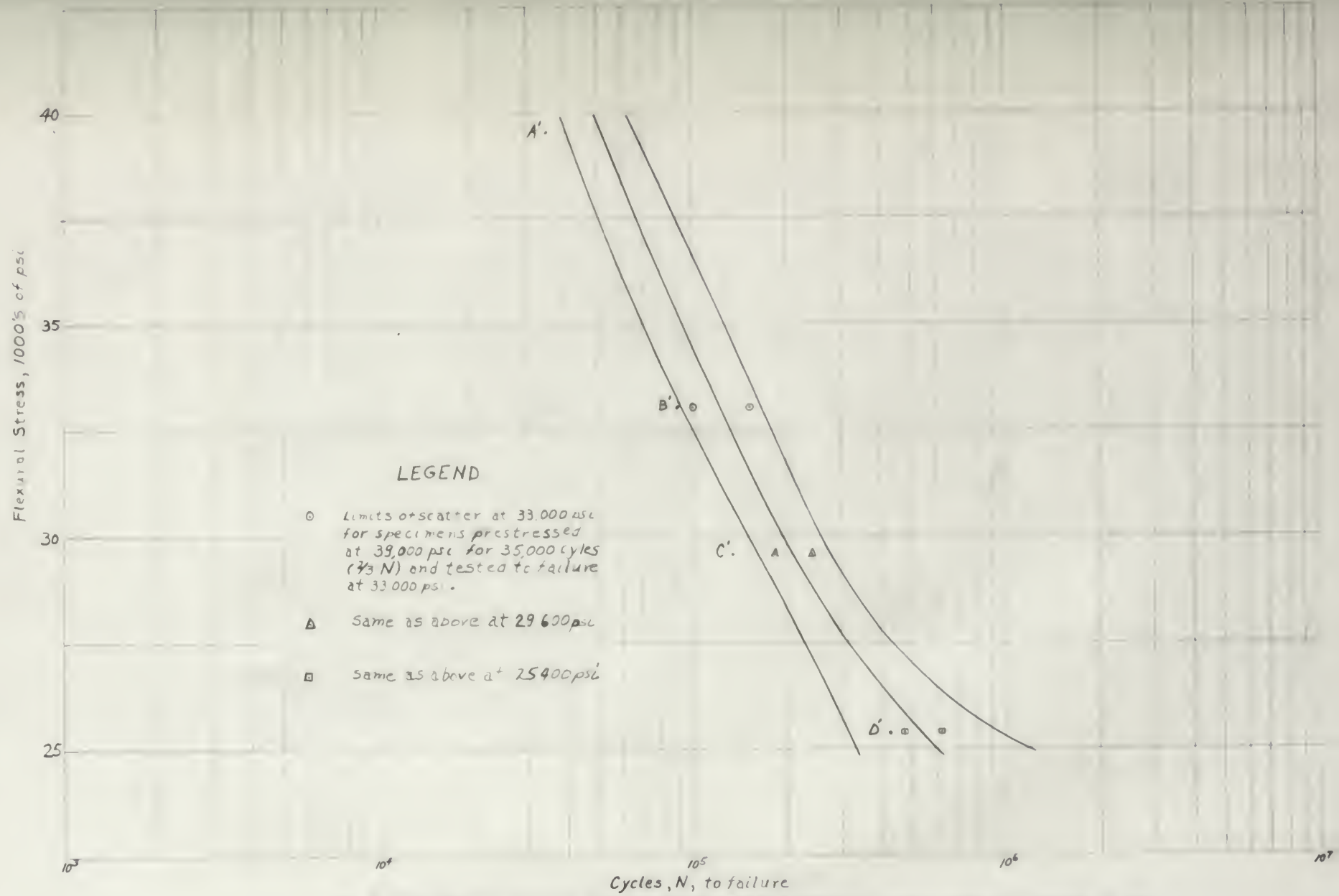


Fig. 5. Plot Showing Damage Points at 33,000, 29,600 & 25,400 psi Corresponding to Damage at ⅓ Fatigue Life at 39,400 psi with Limits of scatter from P=0.05 to P=0.95. (14 tests at each level).

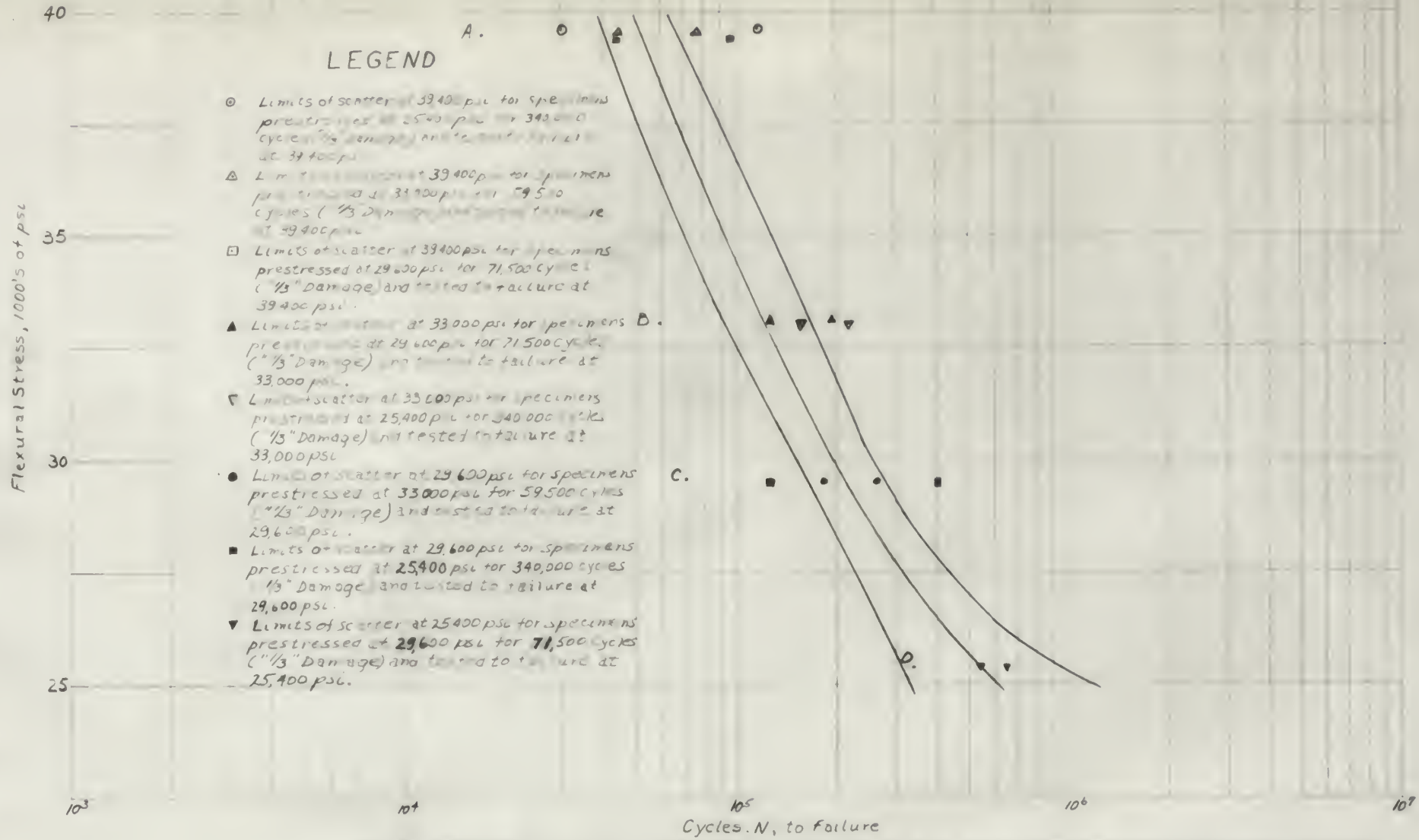

LEGEND
⊙ Limits of scatter at 39,400 psi for specimens prestressed at 25,400 psi for 340,000 cycles ("⅓" Damage) and tested to failure at 39,400 psi.
△ Limits of scatter at 39,400 psi for specimens prestressed at 33,000 psi for 59,500 cycles ("⅓" Damage) and tested to failure at 39,400 psi.
⊡ Limits of scatter at 39,400 psi for specimens prestressed at 29,600 psi for 71,500 cycles ("⅓" Damage) and tested to failure at 39,400 psi.
▲ Limits of scatter at 33,000 psi for specimens prestressed at 29,600 psi for 71,500 cycles ("⅓" Damage) and tested to failure at 33,000 psi.
▽ Limits of scatter at 33,000 psi for specimens prestressed at 25,400 psi for 340,000 cycles ("⅓" Damage) and tested to failure at 33,000 psi.
● Limits of scatter at 29,600 psi for specimens prestressed at 33,000 psi for 59,500 cycles ("⅓" Damage) and tested to failure at 29,600 psi.
■ Limits of scatter at 29,600 psi for specimens prestressed at 25,400 psi for 340,000 cycles ("⅓" Damage) and tested to failure at 29,600 psi.
▼ Limits of scatter at 25,400 psi for specimens prestressed at 29,600 psi for 71,500 cycles ("⅓" Damage) and tested to failure at 25,400 psi.
A.
B.
C.
D.
40
35
30
25
Flexural Stress, 1000's of psi
10^3
10^4
10^5
10^6
10^7
Cycles, N, to Failure

Fig 6 Plot Showing Limits of Scatter from P=0.05 to P=0.95 for Specimens Prestressed to "⅓" Damage Point at one Stress Level and tested to failure at another Stress Level (5 tests at each variation).

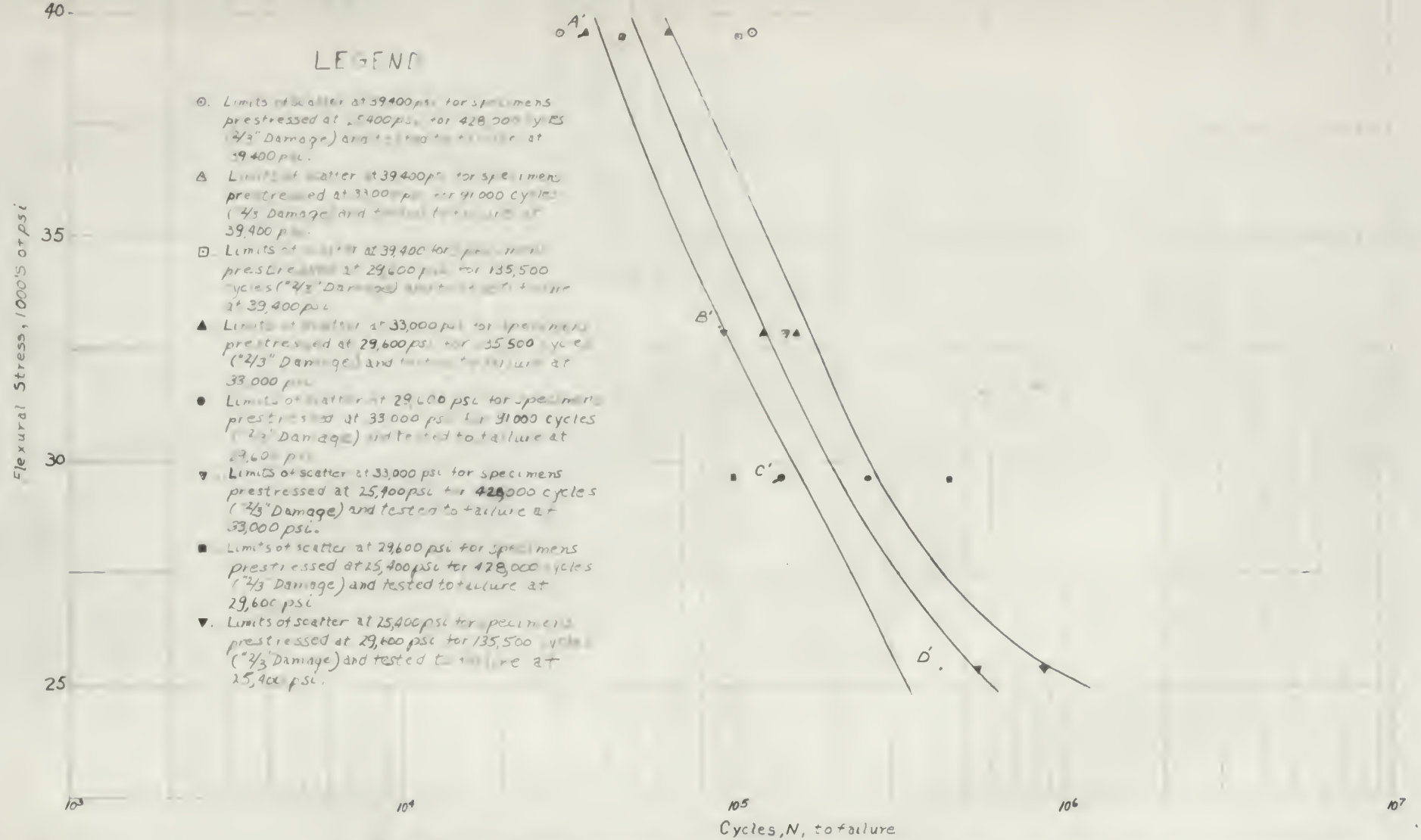

LEGEND
⊙ Limits of scatter at 39,400 psi for specimens prestressed at 25,400 psi for 428,000 cycles ("2/3" Damage) and tested to failure at 39,400 psi.
△ Limits of scatter at 39,400 psi for specimens prestressed at 33,000 psi for 91,000 cycles ("2/3" Damage) and tested to failure at 39,400 psi.
⊡ Limits of scatter at 39,400 for specimens prestressed at 29,600 psi for 135,500 cycles ("2/3" Damage) and tested to failure at 39,400 psi.
▲ Limits of scatter at 33,000 psi for specimens prestressed at 29,600 psi for 35,500 cycles ("2/3" Damage) and tested to failure at 33,000 psi.
● Limits of scatter at 29,600 psi for specimens prestressed at 33,000 psi for 91,000 cycles ("2/3" Damage) and tested to failure at 29,600 psi.
▼ Limits of scatter at 33,000 psi for specimens prestressed at 25,400 psi for 428,000 cycles ("2/3" Damage) and tested to failure at 33,000 psi.
■ Limits of scatter at 29,600 psi for specimens prestressed at 25,400 psi for 428,000 cycles ("2/3" Damage) and tested to failure at 29,600 psi.
▼ Limits of scatter at 25,400 psi for specimens prestressed at 29,600 psi for 135,500 cycles ("2/3" Damage) and tested to failure at 25,400 psi.
A'
B'
C'
D'
40
35
30
25
Flexural Stress, 1000's of psi
10^3
10^4
10^5
10^6
10^7
Cycles, N, to failure

Fig 7. Plot Showing Limits of scatter from P=0.05 to P=0.95 for Specimens Prestressed to "2/3" Damage Point at one stress Level and tested to failure at another stress Level. (5 tests at each variation)

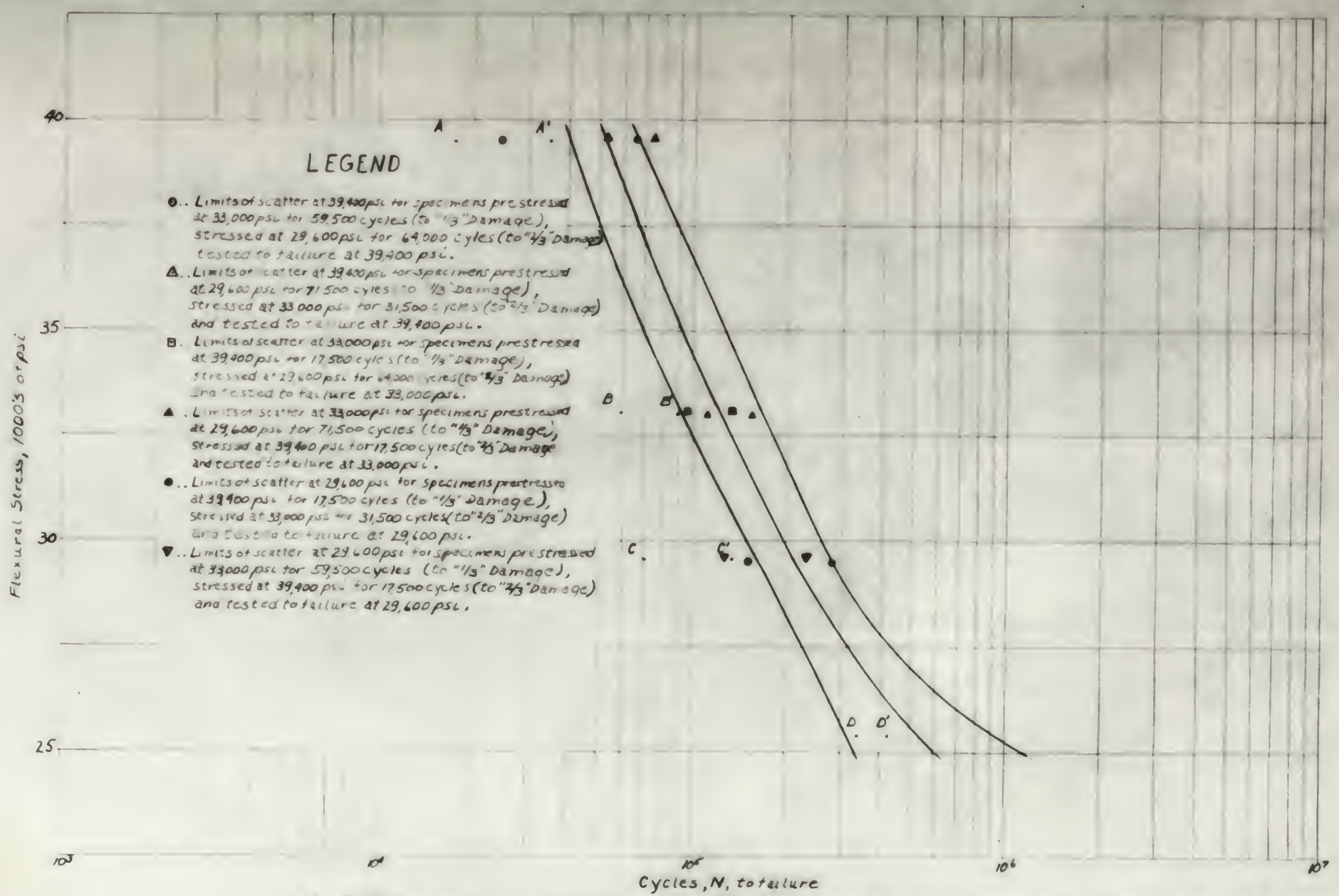

LEGEND
●.. Limits of scatter at 39,400 psi for specimens prestressed at 33,000 psi for 59,500 cycles (to "1/3" Damage), stressed at 29,600 psi for 64,000 cyles (to "2/3" Damage) tested to failure at 39,400 psi.
▲.. Limits of scatter at 39,400 psi for specimens prestressed at 29,600 psi for 71,500 cyles (to "1/3" Damage), stressed at 33,000 psi for 31,500 cycles (to "2/3" Damage) and tested to failure at 39,400 psi.
■. Limits of scatter at 33,000 psi for specimens prestressed at 39,400 psi for 17,500 cyles (to "1/3" Damage), stressed at 29,600 psi for 64,000 cycles (to "2/3" Damage) and tested to failure at 33,000 psi.
▲. Limits of scatter at 33,000 psi for specimens prestressed at 29,600 psi for 71,500 cycles (to "1/3" Damage), stressed at 39,400 psi for 17,500 cyles (to "2/3" Damage and tested to failure at 33,000 psi.
●.. Limits of scatter at 29,600 psi for specimens prestressed at 39,400 psi for 17,500 cyles (to "1/3" Damage), stressed at 33,000 psi for 31,500 cycles (to "2/3" Damage) and tested to failure at 29,600 psi.
▼.. Limits of scatter at 29,600 psi for specimens prestressed at 33,000 psi for 59,500 cycles (to "1/3" Damage), stressed at 39,400 psi for 17,500 cycles (to "2/3" Damage) and tested to failure at 29,600 psi.
A
A'
B
B'
C
C'
D
D'
40
35
30
25
Flexural Stress, 1000's of psi
10^3
10^4
10^5
10^6
10^7
Cycles, N, to failure

Fig 8. Plot - Showing Limits of scatter from P=0.05 to P=0.95 for Specimens Prestressed to "1/3" Damage Point at one Stress Level, stressed to "2/3" Damage Point at another Stress Level and tested to failure at a third stress Level.

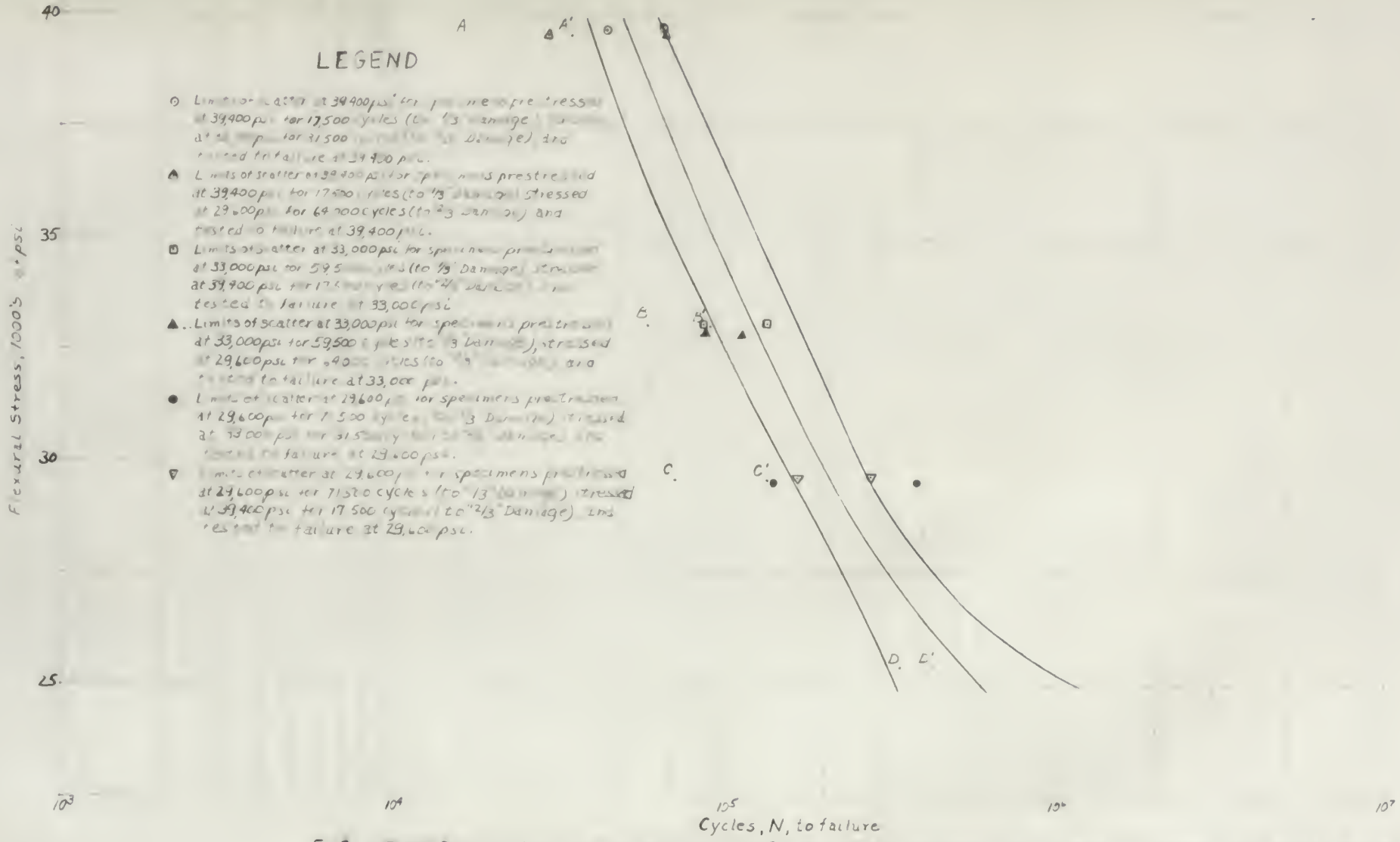


Fig 9. **Plot** - Showing Limits of scatter from P=0.05 to P=0.95 for Specimens Prestress to "1/3" Damage Point at one Stress Level, Stressed to "2/3" Damage Point at another Stress Level and tested to failure at the Original Stress Level (5 tests a each variation).

TABLE 1

TESTS MADE IN ESTABLISHING S-N CURVES

SPECIMEN	MACHINE	STRESS	CYCLES TO FAILURE
1	2	44,600	36,000
2	2	44,000	33,000
3	4	44,600	26,000
4	2	39,400	64,000
5	4	39,400	49,000
6	2	33,000	159,000
7	4	33,000	115,000
8	2	33,000	159,000
9	4	33,000	114,000
10	2	33,000	161,000
11	4	33,000	135,000
12	2	39,400	60,000
13	4	39,400	62,000
14	3	39,400	52,000
15	2	29,600	297,000
16	4	29,600	203,000
17	3	29,600	195,000
18	2	29,600	172,000
19	4	29,600	185,000
20	3	29,600	273,000
21	4	40,500	47,000
22	2	40,500	59,000
23	3	40,500	48,000
24	4	39,400	54,000
25	2	39,400	47,000
26	3	39,400	61,000
27	4	25,400	1,054,000
28	2	25,400	1,015,000
29	2	25,400	613,000
30	4	25,400	669,000
31	1	25,400	496,000
32	2	39,400	49,000
33	4	39,400	53,000
34	1	39,400	48,000
35	2	39,400	71,000
36	4	39,400	86,500
37	1	39,400	55,000
38	2	33,800	99,000
39	4	33,800	80,000
40	1	33,800	114,000
41	2	29,600	211,000
42	1	29,600	219,000
43	4	29,600	218,000
44	4	29,600	212,000
45	2	29,600	227,000
46	1	29,600	199,000
47	2	29,600	170,000
48	1	29,600	215,000
49	4	33,000	115,000
50	2	33,000	146,000
51	1	33,000	110,000
52	4	25,400	512,000
53	1	25,400	597,000
54	2	25,400	572,000
55	4	25,400	296,000
56	3	25,400	375,000
57	1	25,400	501,000
58	2	25,400	517,000
59	4	25,400	406,000
60	1	25,400	363,000
61	4	25,400	465,000
62	2	39,400	525,000
63	2	39,400	51,000
64	4	39,400	53,000
65	1	39,400	41,000
66	2	39,400	53,000
67	2	33,000	131,000
68	1	33,000	128,000
69	4	33,000	143,000
70	2	33,000	91,000
71	1	33,000	97,000
72	4	25,400	442,000
73	1	25,400	585,000

TABLE 2

TESTS MADE IN ESTABLISHING DAMAGE POINTS

SPECIMEN	MACHINE	PRESTRESS	n_p	TESTSTRESS	n_t
4A	4	39,400	17,500	33,000	69,000
5A	2	"	"	"	40,000
6A	1	"	"	"	52,000
7A	4	"	"	"	68,000
8A	2	"	"	"	57,000
9A	4	"	"	"	81,000
10A	2	"	"	"	87,000
11A	1	"	"	"	53,000
12A	4	"	"	"	104,000
13A	2	"	"	"	63,000
14A	1	"	"	"	62,000
15A	2	"	"	"	65,000
16A	4	"	"	"	99,000
17A	1	"	"	"	50,000
18A	4	39,400	17,500	29,600	175,000
19A	2	"	"	"	125,000
20A	1	"	"	"	121,000
21A	2	"	"	"	170,000
22A	4	"	"	"	148,000
23A	4	"	"	"	114,000
24A	2	"	"	"	141,000
25A	4	"	"	"	161,000
26A	1	"	"	"	139,000
27A	2	"	"	"	117,000
28A	4	"	"	"	179,000
29A	1	"	"	"	145,000
30A	2	"	"	"	133,000
31A	1	"	"	"	113,000
51A	4	39,400	17,500	25,400	279,000
52A	2	"	"	"	293,000
53A	1	"	"	"	157,000
54A	1	"	"	"	129,000
55A	4	"	"	"	228,000
56A	2	"	"	"	184,000
63A	4	"	"	"	274,000
64A	1	"	"	"	220,000
65A	2	"	"	"	220,000
20B	4	"	"	"	274,000
21B	2	"	"	"	146,000
22B	1	"	"	"	138,000
39B	4	"	"	"	232,000
40B	2	"	"	"	224,000

SPECIMEN	MACHINE	PRESTRESS	n_p	TESTSTRESS	n_t
57A	1	39,400	35,000	33,000	17,000
58A	2	"	"	"	51,000
59A	4	"	"	"	27,000
60A	4	"	"	"	33,000
61A	1	"	"	"	31,000
62A	2	"	"	"	37,000
66A	4	"	"	"	66,000
67A	2	"	"	"	55,000
68A	1	"	"	"	30,000
69A	1	"	"	"	56,000
70A	4	"	"	"	50,000
1A	1	"	"	"	20,000
2A	4	"	"	"	29,000
3A	2	"	"	"	12,000
71A	2	39,400	35,000	29,600	98,000
72A	4	"	"	"	109,000
73A	1	"	"	"	93,000
74A	2	"	"	"	88,000
75A	4	"	"	"	55,000
76A	1	"	"	"	70,000
77A	2	"	"	"	69,000
78A	4	"	"	"	92,000
79A	2	"	"	"	57,000
80A	1	"	"	"	61,000
81A	4	"	"	"	92,000
82A	2	"	"	"	64,000
83A	1	"	"	"	56,000
84A	4	"	"	"	72,000
7D	4	39,400	35,000	25,400	98,000
8D	2	"	"	"	80,000
9D	3	"	"	"	86,000
10D	4	"	"	"	160,000
11D	2	"	"	"	93,000
12D	3	"	"	"	88,000
13D	1	"	"	"	171,000
14D	2	"	"	"	124,000
15D	3	"	"	"	186,000
16D	4	"	"	"	75,000
17D	1	"	"	"	123,000
18D	2	"	"	"	131,000
19D	3	"	"	"	93,000
20D	4	"	"	"	237,000

TABLE 3

TESTS MADE IN VERIFYING DAMAGE POINTS

TWO STRESS LEVEL TESTS

SP.	M.	PRE-STRESS	n_p	TEST STRESS	n_t	$\Sigma \frac{n}{N}$	PREDICTED $\Sigma \frac{n}{N}$
32A	4	33000	59500	39400	43000	1.286	1.135
33A	2	"	"	"	35000	1.135	1.135
34A	1	"	"	"	33000	1.096	1.135
35A	4	"	"	"	38000	1.192	1.135
36A	2	"	"	"	58000	1.573	1.135
37A	4	29600	71500	39400	30000	0.860	1.005
38A	2	"	"	"	53000	1.348	1.005
39A	1	"	"	"	39000	1.082	1.005
40A	2	"	"	"	69000	1.653	1.005
41A	4	"	"	"	59000	1.463	1.005
42A	1	29600	71500	33000	108000	1.266	0.870
43A	4	"	"	"	124000	1.514	0.870
44A	2	"	"	"	70000	0.889	0.870
45A	1	"	"	"	106000	1.173	0.870
36D	1	"	"	"	98000	1.111	0.870
46A	2	33000	59500	29600	136000	1.076	1.130
47A	4	"	"	"	187000	1.303	1.130
48A	4	"	"	"	162000	1.290	1.130
49A	2	"	"	"	150000	1.138	1.130
50A	1	"	"	"	122000	1.013	1.130
85A	2	33000	91000	39400	9000	0.887	1.051
86A	1	"	"	"	7000	0.849	1.051
87A	4	"	"	"	27000	1.232	1.051
88A	2	"	"	"	15000	1.004	1.051
89A	1	"	"	"	7000	0.851	1.051
90A	2	29600	135500	39400	50000	1.595	0.973
91A	4	"	"	"	29000	1.193	0.973
92A	1	"	"	"	29000	1.193	0.973
93A	4	"	"	"	36000	1.326	0.973
94A	2	"	"	"	12000	0.868	0.973
95A	1	29600	135500	33000	44000	0.986	0.922
96A	4	"	"	"	63000	1.136	0.922
97A	2	"	"	"	79000	1.262	0.922
98A	1	"	"	"	45000	0.994	0.922
99A	4	"	"	"	58000	1.097	0.922
100A	2	33000	91000	29600	77000	1.083	1.078
101A	1	"	"	"	8000	0.756	1.078
102A	1	"	"	"	39000	0.902	1.078
103A	2	"	"	"	96000	1.162	1.078
104A	4	"	"	"	66000	1.030	1.078

SP.	M.	PRE-STRESS	n_p	TEST STRESS	n_t	$\Sigma \frac{n}{N}$	PREDICTED $\Sigma \frac{n}{N}$
21D	4	25400	340000	39400	19000	0.970	1.383
22D	4	"	"	"	60000	1.757	1.383
23D	1	"	"	"	37000	1.131	1.383
24D	2	"	"	"	78000	2.097	1.383
25D	1	"	"	"	38000	1.341	1.381
26D	2	25400	428000	39400	52000	1.767	1.110
27D	4	"	"	"	52000	1.387	1.110
30D	4	"	"	"	14000	1.044	1.110
31D	1	"	"	"	42000	1.577	1.110
32D	2	"	414000			0.752	1.110
28D	4	25400	340000	33000	159000	1.387	1.150
33D	4	"	"	"	136000	1.387	1.150
34D	1	"	"	"	110000	1.572	1.150
35D	2	"	"	"	104000	1.457	1.150
37D	4	"	"	"	140000	1.717	1.150
38D	1	25400	340000	29600	158000	1.412	1.279
39D	2	"	"	"	197000	1.750	1.279
40D	4	"	"	"	84000	1.015	1.279
41D	1	"	"	"	273000	1.947	1.279
42D	2	"	"	"	1000	1.048	1.279
43D	4	25400	411000			0.745	1.137
44D	1	"	428000	29600	218000	1.807	1.137
46D	4	"	386000			0.700	1.137
47D	1	"	428000	29600	197000	1.711	1.137
48D	2	"	368000			0.736	1.137
49D	4	25400	428000	33000	16000	0.903	1.059
50D	1	"	"	"	31000	1.021	1.059
51D	2	"	"	"	24000	1.066	1.059
52D	4	"	"	"	12000	0.879	1.059
53D	1	"	"	"	50000	1.171	1.059
54D	2	29600	71500	25400	248000	0.786	0.742
55D	4	"	"	"	215000	0.730	0.742
56D	1	"	"	"	314000	0.908	0.742
57D	2	"	"	"	237000	0.766	0.742
63D	2	"	"	"	278000	0.843	0.742
58D	4	29600	135500	25400	219000	1.037	0.864
59D	1	"	"	"	274000	1.139	0.864
60D	2	"	"	"	224000	1.048	0.864
61D	4	"	"	"	209000	1.021	0.864
62D	1	"	"	"	463000	1.481	0.864

TABLE 3 (cont'd)

THREE STRESS LEVEL TESTS

SP.	M.	PRE-STRESS	n_p	TEST STRESS(1)	n_{t_1}	TEST STRESS(2)	n_{t_2}	Σr_i	PREDICTED Σr_i
1B	1	39400	17500	33000	31500	29600	35000	0.747	0.941
2B	4	"	"	"	"	"	98000	1.044	0.941
3B	2	"	"	"	"	"	130000	1.196	0.941
4B	1	"	"	"	"	"	61000	0.870	0.941
5B	2	"	"	"	"	"	44000	0.789	0.941
6B	4	39400	17500	29600	64000	33000	94000	0.903	0.917
7B	1	"	"	"	"	"	17000	0.769	0.917
8B	2	"	"	"	"	"	35000	0.911	0.917
9B	4	"	"	"	"	"	[illegible]	0.81[illegible]	.917
10B	1	"	"	"	"	"	8000	0.698	0.917
11B	1	33000	59500	39400	17500	29600	15000	0.872	1.161
12B	4	"	"	"	"	"	73000	1.147	1.161
13B	2	"	"	"	"	"	62000	1.095	1.161
14B	4	"	"	"	17000			0.801	1.161
15B	2	"	"	"	17500	29600	41000	0.995	1.161
16B	1	33000	59500	29600	62000			[illegible]	1.10[illegible]
17B	4	"	"	"	64000	39400	11000	0.980	1.103
18B	2	"	"	"	54000			0.724	1.103
19B	1	"	"	"	64000	39400	1[illegible]00	0.789	1.103
23B	4	"	"	"	"	"	28000	[illegible].303	1.10[illegible]
24B	2	29600	71500	39400	17500	33000	560[illegible]	1.112	0.953
25B	1	"	"	"	"	"	28000	0.891	0.953
26B	1	"	"	"	"	"	30000	0.907	0.953
27B	4	"	"	"	"	"	53000	1.0[illegible]	0.953
28B	2	"	"	"	"	"	44000	1.017	0.953
29B	1	29600	71500	33000	31500	39400	28000	1.065	0.919
30B	4	"	"	"	"	"	23000	1.0[illegible]4	0.919
31B	2	"	"	"	"	"	[illegible]	1.[illegible]54	0.919
32B	1	"	"	"	"	"	22000	1.004	0.919
33B	4	"	"	"	"	"	36000	1.[illegible]72	0.919

SP.	M.	PRE-STRESS	n_p	TEST STRESS(1)	n_{t_1}	TEST STRESS(2)	n_{t_2}	Σr_i	PREDICTED Σr_i
34B	2	39400	17500	33000	31500	39400	21000	0.981	0.914
35B	2	"	"	"	"	"	12000	0.810	0.914
36B	1	"	"	"	"	"	15000	0.866	0.914
37B	4	"	"	"	"	"	22000	1.000	0.914
38B	2	"	"	"	"	"	29000	1.134	0.914
41B	4	33000	59500	39400	17500	33000	26000	1.006	1.083
42B	2	"	"	"	"	"	13000	0.903	1.083
43B	1	"	"	"	"	"	4000	0.833	1.083
44B	2	"	"	"	"	"	8000	0.864	1.083
45B	4	"	"	"	"	"	41000	1.124	1.083
46B	1	33000	59500	29600	64000	33000	20000	0.928	1.052
47B	2	"	"	"	"	"	6000	0.817	1.052
48B	4	"	"	"	"	"	6000	0.817	1.052
49B	4	"	"	"	"	"	13000	0.872	1.052
50B	[illegible]	"	"	"	"	"	3000	0.794	1.052
51B	1	39400	17500	29600	56000			0.598	0.968
52B	1	39400	17500	29600	64000	39400	4000	0.711	0.968
53B	4	"	"	"	"	"	24000	1.093	0.968
54B	2	"	"	"	"	"	16000	0.939	0.968
55B	1	"	"	"	"	"	11000	0.845	0.968
56B	4	29600	71500	39400	17500	29600	125000	1.[illegible]60	1.031
57B	2	"	"	"	"	"	124000	1.259	1.031
58B	1	"	"	"	"	"	64000	0.974	1.031
59B	4	"	"	"	"	"	75000	1.026	1.031
61B	2	"	"	"	"	"	61000	0.955	1.031
60B	1	29600	71500	33000	31500	39600	47000	0.808	0.945
62B	4	"	"	"	"	"	98000	1.049	0.945
63B	2	"	"	"	"	"	69000	0.912	0.945
64B	4	"	"	"	"	"	145000	1.272	0.945
65B	[illegible]	"	"	"	"	"	74000	[illegible].936	[illegible].945

TABLE 4

STANDARD DEVIATION AND MEAN LOG N FOR TESTS

MADE IN ESTABLISHING THE S-N CURVES

STRESS	NO. OF TESTS	MEAN LOG N	STANDARD DEVIATION
39400	14	4.725	0.0600
33000	14	5.104	0.0800
29600	14	5.325	0.0670
25400	14	5.741	0.1420

TABLE 5

STANDARD DEVIATION AND MEAN LOG N FOR TESTS

MADE IN ESTABLISHING "1/3" DAMAGE POINTS

PRESTRESS	TEST STRESS TO FAILURE	NO. OF TESTS	MEAN LOG N	STANDARD DEVIATION
39400	33000	14	5.104	0.0623
39400	29600	14	5.325	0.0455
39400	25400	14	5.741	0.0440

TABLE 6

STANDARD DEVIATION AND MEAN LOG N FOR TESTS

MADE IN ESTABLISHING "2/3" DAMAGE POINTS

PRESTRESS	TEST STRESS TO FAILURE	NO. OF TESTS	MEAN LOG N	STANDARD DEVIATION
39400	33000	14	5.104	0.0555
39400	29600	14	5.325	0.0332
39400	25400	14	5.741	0.0367

TABLE 7

SHOWING STANDARD DEVIATION AND MEAN LOG N

ENCOUNTERED IN TESTING DAMAGE POINTS

TWO STRESS LEVEL TESTS

PRESTRESS TO "1/3" DAMAGE	TEST STRESS TO FAILURE	NO. OF TESTS	MEAN LOG N	STANDARD DEVIATION
33,000	39,400	5	4.766	0.0695
33,000	29,600	5	5.346	0.0481
33,000	25,400	5	5.776	0.0248
29,600	39,400	5	4.820	0.1402
29,600	33,000	5	5.201	0.0563
25,400	39,400	5	4.765	0.1705
25,400	33,000	5	5.281	0.0478
25,400	29,600	5	5.351	0.1510

PRESTRESS TO "2/3" DAMAGE	TEST STRESS TO FAILURE	NO. OF TESTS	MEAN LOG N	STD. DEV.
33,000	39,400	5	4.677	0.0714
33,000	29,600	5	5.278	0.0815
33,000	25,400	5	5.845	0.0609
29,600	39,400	5	4.853	0.1035
29,600	33,000	5	5.171	0.0416
25,400	39,400	5	4.769	0.1720
25,400	33,000	5	5.069	0.0538
25,400	29,600	5	5.280	0.2320

TABLE 7 (cont'd)

THREE STRESS LEVEL TESTS

PRESTRESS TO 1/3 DAMAGE	STRESS TO 2/3 DAMAGE	STRESS TO FAILURE	NO. OF TESTS	MEAN LOG N	STANDARD DEVIATION
39400	33000	29600	5	5.314	0.0809
39400	29600	33000	5	5.057	0.0438
39400	33000	39400	5	4.736	0.0522
39400	29600	39400	5	4.645	0.1078
33000	39400	33000	5	5.036	0.0578
33000	39400	29600	5	5.233	0.0790
33000	29600	39400	5	4.605	0.1275
33000	29600	33000	5	5.002	0.0325
29600	39400	33000	5	5.123	0.0424
29600	39400	29600	5	5.348	0.0615
29600	33000	39400	5	4.799	0.0458
29600	33000	29600	5	5.368	0.1315

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